Differentiation IGCSE (0580)

Objectives:-

→ Defining Differentiation ✓
→ Ways of Expressing ✓
→ Rules ✓
→ find gradient ✓
→ finding stationary / Turning Point ✓
→ finding Maximum and Minimum point ✓
→ Past Paper Questions

Syllabus:

→ Using the power rule
→ finding slope (gradient) at any x-value.
→ Spot flat points (Turning, Stationary point) on a curve
→ Discriminate between maximum and minimum point.
→ Equation of tangent

Definition:-

It is a measure of rate of change of a function with respect to its input.

Expressing:

function	Derivative
y →	$\frac{dy}{dx}$
$f(x)$ →	$f'(x)$

Rules:

1) $y = ax^n$ (a: constant, n: Power, x: variable)

$\boxed{\frac{dy}{dx}} = nax^{n-1}$

2) $\boxed{y = ax}$ (a: constant, x: variable)

$\boxed{\frac{dy}{dx} = a}$

Example:

1) $y = 3x^4$

$\frac{dy}{dx} = 4(3)x^{4-3}$

$= 12x^3$

2) $y = 7x^1$

$\frac{dy}{dx} = 1(7)x^{1-1}$

$= 7x^0$

$= 7(1)$

$= 7$

* x^0 / variable to the power of zero → 1

$y = 8x^1$

$= 8x^{1-1}$

$= 8x^0$

$8(1) = 8$

3) $\boxed{y = a}$ → constant

$\boxed{\frac{dy}{dx} = 0}$

$\boxed{y = 8}$

$\frac{dy}{dx} = 8(x^0)$

$= 0(8)(x^{0-1})$

$= 0$

Finding gradient:-

Rule:

Take derivative of the given equation of line. After that place the 'x' coordinate in the resulting equation to find the gradient.

Example:-

Find the gradient of the curve $y = x^3$ at the point $(2, 8)$

$y = x^3$

$\frac{dy}{dx} = 3x^{3-1}$

$= \boxed{3x^2}$

$= 3(2)^2$

$= \boxed{12}$

$(\overset{x}{2}, \overset{y}{8})$

Turning Point / Stationary Point:-

Rule:-

Take the derivative of the given equation and put it equal to zero to find the value of 'x'. Place the value of 'x' in original equation to find 'y'.

Example:-

Find the Turning point of the equation $\boxed{y = 2x^2 - 8x + 5.}$

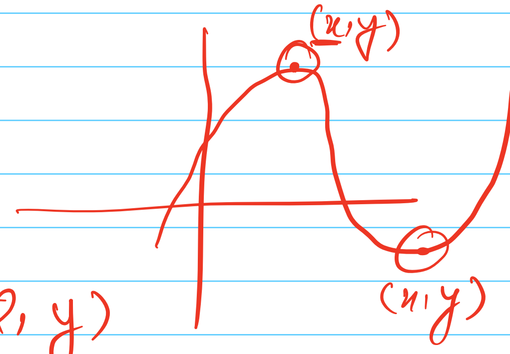
$$y = 2x^{2} - 8x^{1} + 5$$

$$\frac{dy}{dx} = 2(2)x^1 - 8\underset{(1)}{x^0} + 0$$

$$4x - 8 = 0$$

$$4x = 8$$

$$\boxed{x = 2}$$



$(2, y)$

$(2, -3)$

$$y = 2x^2 - 8x + 5$$

$$= 2(2)^2 - 8(2) + 5$$

$$= 8 - 16 + 5$$

$$= \boxed{-3}$$

Finding whether Stationery Point is maximum or minimum:-

Rule:-

1) Put the derivative equal to zero and find the two values of 'x'.

2) Place the two found values of 'x' in original equation one after the other to find the two values of 'y'! This will result in two points.

3) Take double derivative. If:-

found answer < 0 (Maximum Point)

found answer > 0 (Minimum Point).

Example:-

Find the maximum or minimum point of the equation $y = x^3 - 12x^2 + 21x$. Determine whether the turning point is maximum or minimum.

$y = x^3 - 12x^2 + 21x$

$\frac{dy}{dx} = \boxed{3x^2 - 24x + 21}$

$3x^2 - 24x + 21 = 0$

$3(x^2 - 8x + 7) = 0$

$x^2 - 8x + 7 = 0$

$x^2 - 7x - x + 7 = 0$

$x(x-7) - 1(x-7) = 0$

$(x-1)(x-7) = 0$

either $x - 1 = 0$ or $x - 7 = 0$

$\boxed{x = 1 \qquad x = 7}$

$y = x^3 - 12x^2 + 21x$

$y = (1)^3 - 12(1)^2 + 21(1)$

$= 1 - 12 + 21 = +10$

$y = x^3 - 12x^2 + 21x$

$y = (7)^3 - 12(7)^2 + 21(7)$

$= 343 - 588 + 147 = \boxed{-98}$

$(1, 10)$ Maximum $\qquad (7, -98)$ Minimum.

$\frac{d^2y}{dx^2} = 3x^2 - 24x + 21$

$= \boxed{6x - 24}$

$6(1) - 24 = -18 < 0$

$6x - 24$

$6(7) - 24$

$42 - 24 = +18 > 0$

Equation of line of a tangent:-

Rule:-

→ Equation of line is $\boxed{y = mx + c}$ → m: $\boxed{\text{gradient}}$, c: $\boxed{\text{y-Intercept}}$

→ Gradient will be found by taking derivative and after that place the given coordinate to find 'c'.

Example:-

Find the equation of tangent at point $\boxed{(5, 8)}$ of the curve $\boxed{y = 4x^2 + 5x + 6}$

$\frac{dy}{dx} = 4x^2 + 5x + 6 \to 0$

$= \boxed{8x + 5}$

$= 8(5) + 5 = 40 + 5 = 45$

$y = mx + c$

$y = 45x + c$

$\boxed{y = 45x - 217}$

$y = 45x + c$

$8 = 45(5) + c$

$8 = 225 + c$

$\boxed{-217} = 8 - 225 = c$

Past Paper Questions (Oct Nov P4 2024)

10 A curve has the equation $y = x^3 - 9x^2 - 48x$.

(a) Differentiate $x^3 - 9x^2 - 48x$.

$$\frac{dy}{dx} = \boxed{3x^2 - 18x - 48}$$

$3x^2 - 18x - 48$ .... [2]

(b) Find the coordinates of the turning points of the graph of $y = x^3 - 9x^2 - 48x$.
You must show all your working.

$y = x^3 - 9x^2 - 48x$

$\frac{dy}{dx} = \boxed{3x^2 - 18x - 48}$

$3x^2 - 18x - 48 = 0$

$3(x^2 - 6x - 16) = 0$

$x^2 - 6x - 16 = 0$

$x^2 - 8x + 2x - 16 = 0$

$x(x-8) + 2(x-8) = 0$

$(x+2)(x-8) = 0$

either $\boxed{x = -2}$ or $\boxed{x = 8}$

$x = -2$

$y = x^3 - 9x^2 - 48x$

$y = (-2)^3 - 9(-2)^2 - 48(-2)$

$= -8 - 36 + 96$

$= 52$

$(-2, 52)$

$\boxed{x = 8}$

$y = 8^3 - 9(8)^2 - 48(8)$

$y = 512 - 576 - 384$

$= \boxed{-448}$

(-2 , 52) and (8 , -448)

[4]

(c) Determine whether each of the turning points is a maximum or a minimum.
Give reasons for your answers.

$3x^2 - 18x - 48$

$\frac{d^2y}{dx^2} = 6x - 18$

$= 6(-2) - 18$

$= -12 - 18$

$= -30 < 0$

(Maximum point

$3x^2 - 18x - 48$

$= 6x - 18$

$= 6(8) - 18$

$= 48 - 18$

$= 30 > 0$

[3]

$(-2, 52)$ Maximum point

$(8, -448)$ Minimum point

11 **(a)** The point $(-1, 6)$ lies on a curve.

This curve has the derived function $\frac{dy}{dx} = -4x^3 - 9x^2 + 5$.

Show that $(-1, 6)$ is a stationary point of the curve.

[2]

(b) A different curve has equation $y = 2x^3 - 6x + 8$.

(i) Calculate the gradient of the tangent to this curve at the point $(-2, 2)$.

.... [3]

(ii) Find the x-coordinates of the stationary points of this curve.

$x =$ .... and $x =$ .... [2]